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EWPT Nucleation With MSSM and Electromagnetic Field Creation

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Using EW-MSSM field theory, so the EWPT is first order, we derive the equations of motion for the gauge fields. With an isospin ansatz we derive e.o.m. for the electrically charged W fields uncoupled from all other fields. These and the lepton currents serve as the current for the Maxwell-like e.o.m. for the electromagnetic field. The electromagnetic field arising during EWPT bubble nucleation without leptons is found. We then calculate the electron current contribution, which is seen to be quite large. This provides the basis for determinining the magnetic field created by EWPT bubble collisions, which could seed galactic and extra-galactic magnetic fields.

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I. INTRODUCTION

There have been many attempts to find the origin of the large-scale galactic and extra-galactic magnetic fields that have been observed (see Ref[1] for a review). One possible seeding of these magnetic fields is via early universe phase transitions: the electroweak phase transition (EWPT) and the Quantum Chrodynamics chiral phase transition (QCDPT). Research on the QCDPT has explored not only seeding of the galactic and extra-galactic structure, but also possible effects in Cosmic Microwave Background Radiation (CMBR) correlations[2–11]; and large scale magnetic structure which could produce observable CMBR polarization correlations[12] arising from bubble collisions during the QCDPT[13].

Since the electromagnetic (em) field along with the $W^{\pm}$ and Z fields are the gauge fields of the Standard EW model, the EWPT is particularly interesting for exploring possible cosmological magnetic seeds . It has been shown that in the Standard EW model there is no first order EWPT[14], no explanation of baryogenesis, or any interesting cosmological magnetic structures created during the crossover transition. However, there has been a great deal of activity in the supersymmetric extension of the standard model[15], and with a MSSM having a Stop with a mass similar to the Higgs there can be a first order phase transition and consistency with baryogenesis[16–18].

Moreover, with a first order phase transitions bubbles of the new phase nucleate and collide, which is essential for our present research on magnetic field creation. A number of authers have used EW-MSSM models with the Standard EW Model plus a right-handed Stop to explore the EWPT. See, e.g., Refs. [16, 19]. There have been other models for CP violation and baryogenesis, such as two-Higgs models (see Refs.[16–18] for references and discussion) and leptoquarks (see, e.g., Ref.[20] for a discussion and references.)

The goal of our present work on the creation of electromagnetic fields via nucleation is both to find the nature of the fields and to derive the boundary conditions for our study of magnetic field generation with EWPT bubble collision, which is in progress[21, 22]. For this reason we introduce the complete Lagrangian that is needed to study bubble collisions. In our present calculations we could use a two-Higgs model, as the precise nature of the extension of the Standard EW Model is not needed for creation of electromagnetic fields during nucleation, as we shall show; while in our studies of magnetic field generation with EWPT bubble collision we use the MSSM and treat all the equations of motion, derived below, with a right-handed Stop.

In the most detailed previous research on the study of magnetic fields arising from EWPT transitions[8–10] a model was used in which the equations of motion (e.o.m.) for the em field involved the chargeless Higgs, and the EWPT bubbles were empty of the em field until the bubbles collided and overlapped. In the present work we derive the e.o.m. directly from the EW Lagrangian, using a MSSM including the right-handed Stop field for consistency with a first order phase transition.

The e.o.m. which we obtain for the em field is Maxwell-like, with the current given by the charged $W^{\pm}$ fields, which is physically reasonable.

In general, the e.o.m. are complicated coupled partial differential equations, in which solutions for the Higgs and Stop fields must be found to obtain the current for the em field equation. In the present paper, however, we introduce an I-spin formulation which allows us to uncouple the e.o.m. for the $W^{\pm}$ fields, and carry out a study of spherically symmetric EW bubble nucleation. For pure nucleation of a bubble we first solve the $W^{\pm}$ e.o.m. numerically without leptons, and obtain solutions for the em field using a fit to these numerical solutions for the $W^{\pm}$ fields. Next we include the electron current and solve the new equations for the electromagnetic field.

The solutions have an instanton-like form near the bubble wall, as expected. These are the first solutions for the electromagnetic field from EWPT nucleation that have been obtained from the EW Lagrangian. Due to spherical symmetry during nucleation only electric fields are produced. These give the initial fields for bubble collisions. For the derivation of EW cosmological magnetic seeds, the more complicated collision problem must also be solved[21]

In Section II we present the EW-MSSM model, including fermions, and derive the e.o.m. for the gauge fields and the Higgs and Stop fields. In Section III the I-Spin ansatz is used to separate the gauge fields, and we present the solutions for the em field without fermions. In Section IV we include electrons, the mmost important fermion for em field creation, and find the em field. Our conclusions are given in Section V

II. MSSM EW EQUATIONS OF MOTION WITH RIGHT-HANDED STOP

In this section we derive the equations of motion for the standard Weinberg-Salam EW model extended to a MSSM model with the addition of a Stop field, the partner to the top quark. all other partners of the standard model fields are integrated out.

A. MSSM EW Lagrangian with Fermions

$$\mathcal{L}^{MSSM} = \mathcal{L}^1 + \mathcal{L}^2 + \mathcal{L}^3 + \mathcal{L}^{fermion} \tag{1}$$

$$\begin{aligned}
\mathcal{L}^1 &= -\frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
\mathcal{L}^2 &= |(i\partial_\mu - \frac{g}{2}\tau\cdot W_\mu - \frac{g'}{2}B_\mu)\Phi|^2 - V(\Phi) \\
\mathcal{L}^3 &= |(i\partial_\mu - \frac{g_s}{2}\lambda^a C^a_\mu)\Phi_s|^2 - V_{hs}(\Phi_s,\Phi) \\
\mathcal{L}^{fermion} &= \text{standard Lagrangian for fermions}\ ,
\end{aligned}$$

where the pure C^a_μ term is omitted in $\mathcal{L}^1$ and

$$\begin{aligned}
W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\ ,
\end{aligned} \tag{2}$$

where the W^i, with i = (1,2), are the W^+, W^- fields, C^a_μ is an SU(3) gauge field, (Φ, Φ_s) are the (Higgs, right-handed Stop fields), (τ^i, λ^a) are the (SU(2),SU(3) generators, and the electromagnetic and Z fields are defined as

$$\begin{aligned}
A^{em}_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(g'W^3_\mu + gB_\mu) \\
Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gW^3_\mu - g'B_\mu)\ .
\end{aligned} \tag{3}$$

The standard fermion Lagrangian, $\mathcal{L}^{fermion}$ has the form

$$\begin{aligned}
\mathcal{L}^{fermion} &= \sum_j [\bar{\psi}^j_L\gamma^\mu(i\partial_\mu - \frac{g}{2}\tau\cdot W_\mu - g'\frac{Y}{2}B_\mu)\psi^j_L \\
&\quad + \bar{\psi}^j_R\gamma^\mu(i\partial_\mu - g'\frac{Y}{2}B_\mu)\psi^j_R] \\
&\quad + \sum_c f^i\bar{\psi}^c_L\Phi\psi^c_R\ ,
\end{aligned} \tag{4}$$

with j representing all fermions, while the sum over c is just for charged fermions. The first term in $\mathcal{L}^{fermion}$ gives the interaction of fermions with the gauge fields, while second term corresponds to the Higgs EWPT creation of fermion masses. We neglect these masses, as well as fermion masses created by the Stop, in the present work.

B. MSSM EW Equations of Motion Without Fermions

The effective Higgs and Stop potentials are taken as

$$\begin{aligned}
V(\Phi) &= -\mu^2|\Phi|^2 + \lambda|\Phi|^4 \\
V_{hs}(\Phi,\Phi_s) &= -\mu_s^2|\Phi_s|^2 + \lambda_s|\Phi_s|^4 \\
&\quad + \lambda_{hs}|\Phi|^2|\Phi_s|^2\ .
\end{aligned} \tag{5}$$

The various parameters are discussed in many publications[16]. In particular we need $g = e/sin\theta_W = 0.646$, $g' = g\ tan\theta_W = 0.343$, and $G = gg'/\sqrt{g^2+g'^2} = 0.303$.

In the picture we are using, the Higgs and Stop fields will play a dynamic role in the EW bubble nucleation and collisions, and we shall need the space-time structure of these fields rather than only the vacuum expectation value for a particular vacuum state for the complete solutions of the e.o.m. Our form for the Higgs field, Φ, is

$$\Phi(x) \;=\; \begin{pmatrix} 0 \\ \phi(x) \end{pmatrix} . \tag{6}$$

and

$$\tau \cdot W_\mu \Phi \;=\; \begin{pmatrix} (W^1_\mu - iW^2_\mu) \\ -W^3_\mu \end{pmatrix} \phi(x) . \tag{7}$$

In the present exploratory paper treating bubble nucleation we center on the possible generation of an electromagnetic field, and the solution of all of the e.o.m. is avoided. Therefore, specific forms and solutions for the Higgs and Stop fields do not enter the equations needed for the present work. For this reason we do not choose a specific form for the right-handed Stop field, Φ_s, and for convenience write the additional MSSM gauge field as

$$C_\mu \;=\; \frac{\lambda^a}{2} C^a_\mu . \tag{8}$$

We also use the definitions

$$\begin{aligned} \phi(x) &\equiv \rho(x) e^{i\Theta(x)} \\ |\phi(x)|^2 &= \rho(x)^2 \\ |\Phi_s(x)|^2 &\equiv \rho_s(x)^2 . \end{aligned} \tag{9}$$

Although we do not need specific forms for C_μ or Φ_s, we assume that a Stop condensate is formed for consistency with a first order EWPPT, as in Ref [19]

With these definitions $\mathcal{L}^2$ is (j = (1,2,3))

$$\begin{aligned} \mathcal{L}^2 \;=\; & \partial_\mu \phi^* \partial^\mu \phi + (i(\partial_\mu \phi^*)\phi - i\phi^* \partial_\mu \phi)(-gW^3_\mu \\ & + g' B_\mu)/2 + \phi^*\phi((\frac{g}{2})^2 (W^j)^2 + (\frac{g'}{2})^2 B^2 \\ & - \frac{gg'}{2} W^3 \cdot B) - V(\phi) . \end{aligned} \tag{10}$$

The equations of motion for the charged gauge fields, W^i, for i=(1,2) are obtained by minimizing the action

$$\delta \int d^4x [\mathcal{L}^1 + \mathcal{L}^2 + \mathcal{L}^3] \;=\; 0 , \tag{11}$$

i.e., we do not include $\mathcal{L}^{fermion}$. The equations of motion that we obtain from the variations in W^i, for i=(1,2) are

$$\begin{aligned} & \partial^2 W^i_\nu - \partial^\mu \partial_\nu W^i_\mu - g\epsilon^{ijk} \mathcal{W}^{jk}_\nu + \frac{g^2}{2}\rho(x)^2 W^i_\nu \\ & = 0 , \end{aligned} \tag{12}$$

with

$$\begin{aligned} \mathcal{W}^{jk}_\nu &\equiv \partial^\mu (W^j_\mu) W^k_\nu + W^j_\mu \partial^\mu W^k_\nu + W^{j\mu} W^k_{\mu\nu} , \\ W^k_{\mu\nu} &= \partial^\mu W^k_\nu - \partial^\nu W^k_\mu - g\epsilon^{klm} W^l_\mu W^m_\nu \end{aligned} \tag{13}$$

The e.o.m. for A^{em}, Z, including the eletron em current, J^e_ν, are

$$\begin{aligned} & \partial^2 A^{em}_\nu - \partial_\mu \partial_\nu A^{em}_\mu - \frac{gg'}{\sqrt{g^2+g'^2}} \epsilon^{3jk} \mathcal{W}^{jk}_\nu \\ & -J^e_\nu \quad = 0 \end{aligned} \tag{14}$$

$$\begin{aligned} & \partial^2 Z_\nu - \partial_\mu \partial_\nu Z_\mu - \frac{\rho^2 \partial_\nu \Theta}{\sqrt{g^2+g'^2}} \\ & - \frac{g^2}{\sqrt{g^2+g'^2}} \epsilon^{3jk} \mathcal{W}^{jk}_\nu = 0 . \end{aligned} \tag{15}$$

Note that we have dropped all fermion currents except the electron current.

The e.o.m. for the Higgs field are

$$\begin{aligned} & \frac{1}{\rho(x)} \partial^2 \rho(x) - \mu^2 + 2\lambda \rho(x)^2 + \lambda_{hs} \rho_s(x)^2 - H \cdot H \\ & - \partial_\mu \Theta \partial^\mu \Theta + \frac{\sqrt{g^2+g'^2}}{2} Z^\mu \partial_\mu \Theta = 0 , \end{aligned} \tag{16}$$

with

$$H \cdot H \;\equiv\; (\frac{g}{2})^2 W^i \cdot W^i + (\frac{g'}{2})^2 B \cdot B - \frac{gg'}{2} W^3 \cdot B ,$$

and

$$\partial_\mu (\rho(x)^2 \partial^\mu \Theta - \frac{\sqrt{g^2+g'^2}}{2} \rho(x)^2 Z^\mu) \;=\; 0 . \tag{17}$$

The e.o.m. for the right-handed Stop is

$$\begin{aligned} & -\partial^2 \Phi_s + ig_S(\partial^\mu (C_\mu \Phi_s) + (C_\mu \partial^\mu \Phi_s)) + (g_s^2 C^\dagger_\mu C_\mu \\ & + \mu_s^2 + 2\lambda_s \rho_s^2 + \lambda_{hs} \rho^2) \Phi_s = 0 , \end{aligned} \tag{18}$$

We do not give the e.o.m. for the C_μ gauge field, which is not needed in the present work.

These are exact equations of motion in our MSSM model with a right-handed stop, without fermions,

which (as we shall see in the next Section) have only small effects In our research on EWPT collisions, where we calculate the magnetic field produced by the EWPT, we need the equations for the Higgs and Stop fields, but in our present work on nucleation we only calculate the gauge fields, with the inlusion of the eletron current given in the following section.

III. I-SPIN ANSATZ AND ELECTROMAGNETIC FIELD CREATION FROM $W^{\pm}$

One of the most important features of the equations of motion derived directly from the EW Lagrangian is that the source current of the electromagnetic field is given by the charged gauge $W^{\pm}$ fields, as seen from the Maxwell-like Eq.(14). This is expected physically, and is in sharp contrast with the Kibble-Vilenkin[8], Ahonen-Enqvist[9], Copeland-Saffin-Törnkvist[10] picture. This suggests that we use an SU(2), isospin ansatz for the gauge fields.

A. I-Spin Ansatz

In the present paper we assume that

$$\begin{aligned} W^j_\nu &\simeq i\tau^j W_\nu(x) \simeq i\tau^j x_\nu W(x) \;\; \mathrm{j}=1,2,3 \\ A^{em}_\nu &\simeq i\tau^3 A_\nu(x) \simeq i\tau^3 x_\nu A(x) \\ Z_\nu &\simeq i\tau^3 Z(x)_\nu \simeq i\tau^3 x_\nu Z(x) \;, \end{aligned} \tag{19}$$

with the I-spin operators defined as $\epsilon^{mjk}\tau^j\tau^k = i\tau^m$. We shall see that this enables us to derive the straight-forward equations of motion for the electromagnetic field, which can be solved to a good approximation for symmetric nucleation of EW bubbles. In this section we derive the e.o.m. for spherically symmetric bubble nucleation, so that $W(x) = W(r,t)$ and $A(x) = A(r,t)$, with $x^\mu x_\mu = t^2 - r^2$. First note that

$$\epsilon^{ijk}\mathcal{W}^{jk}_\nu \;=\; i\tau^i \times F[W_\nu, \partial_\nu W] \;, \tag{20}$$

with F a function of W_ν and $\partial_\nu W$ to be determined, so that the e.o.m. for W_ν, Eq.(12), becomes

$$\begin{aligned} &\partial^\mu\partial_\mu W_\nu - \partial_\nu\partial_\mu W^\mu - g x_\nu[5W^2 + \\ &3W(t\partial_t + r\partial_r)W + gs^2W^3 - \beta s^2\partial_r W] \\ &-\frac{g^2}{2}\rho^2 W_\nu = 0 \;, \end{aligned} \tag{21}$$

with $W_\nu = x_\nu W(r,t)$, $s^2 = t^2 - r^2$, $r = \sqrt{\sum_{j=1}^3 x^j x^j} = \sqrt{-\sum_{j=1}^3 x^j x_j}$, $\partial_j r = x^j/r$, and $\beta = (+,-)$ for $\nu = (t,j)$. Subtracting the e.o.m. for $W_t \times x_j$ from the e.o.m. for $W_j \times t$ we find

$$\begin{aligned} &(\partial_t^2 + \partial_r^2)W + \frac{t^2+r^2}{rt}\partial_t\partial_r W + (\frac{3}{r}\partial_r + \frac{3}{t}\partial_t)W \\ &+gW(t^2-r^2)(\frac{1}{t}\partial_t - \frac{1}{r}\partial_r)W = 0 \end{aligned} \tag{22}$$

$$\begin{aligned} &(\partial_t^2 + \partial_r^2)A + \frac{t^2+r^2}{rt}\partial_t\partial_r A + (\frac{3}{r}\partial_r + \frac{3}{t}\partial_t)A \\ &+GW(t^2-r^2)(\frac{1}{t}\partial_t - \frac{1}{r}\partial_r)W = 0 \;. \end{aligned} \tag{23}$$

The most significant aspect of the I-spin formulation is that we obtain e.o.m. for $W(r,t)$ and for $A(r,t)$ without contributions from the Higgs or Stop fields.

As pointed out at the beginning of this section, the current for the electromagnetic field arises entirely form the electrically charged fields/particles, $W^{\pm}$, as seen also in Eq.(23). Moreover, the current within our I-spin formulation is determined by a nonlinear partial equation for $W(r,t)$, without direct coupling to the Higgs or Stop fields. This will enable us to derive the electromagnetic fields from EWPT bubble collisions as in Refs.[8–10]. In the present paper, however, we derive the electromagnetic fields produced in the EWPT via bubble nucleation before collisions, which has not been considered previously. For collisions a direction in space is singled out, so that the form $W(r,t), A(r,t)$ cannot be used. This is a topic for future work. Also, fermions contribute to the electric current, and fermion fields will be included in future work. The solution for A^{em} produced during nucleation with the assumptions of the present section are found in the following section.

B. I-Spin Ansatz and Electromagnetic Field Creation During Nucleation Without Fermions

In the present work we make use of the gauge fields gauge conditions to reduce the partial differential equations, Eqs(22,23), to ordinary differential equations. The philosophy is to derive the $W^{\pm}$ and A^{em} fields as a function of r at a fixed time. Since from the general structure of the equations we expect at time t that the bubble wall will be at $r = r_w \simeq t$, we are mainly interested in the nature of the fields near r = r_w. As we shall see, since the solutions are modified instanton-like in nature, the most significant region for magnetic field creation for both nucleation, and for collisions, will be at the bubble walls.

We use the Coulomb gauge, which is consistent with spherical spatial symmetry, giving the equations for $W(r,t)$ and $A(r,t)$:

$$\begin{aligned} \sum_{j=1}^{3} \partial_j W^j &= \sum_{j=1}^{3} \partial_j A^j = 0 \ \text{ or} \\ r\partial_r W(r,t) + 3W(r,t) &= 0 \,, \end{aligned} \tag{24}$$

with solutions for each value of r

$$W(r,t) = \frac{W_r(t)}{r^3} \quad A(r,t) = \frac{A_r(t)}{r^3} \,. \tag{25}$$

From Eq.(22) and the gauge condition (24), one obtains differential equations for $W^t(r,t)$ and $W^j(r,t)$. By combining them one finds that the Higgs and Stop fields are disconnected, and obtains an e.o.m. for the functions $W_r(t)$ and $A_r(t)$

$$\begin{aligned} W_r''(t) - \frac{3t}{r^2}W_r'(t) + \frac{3}{r^2}W_r(t) + g\frac{t^2-r^2}{r^3}W_r(t) \\ (\frac{1}{t}W_r'(t) - \frac{3}{r^2}W_r(t)) &= 0 \end{aligned} \tag{26}$$

$$\begin{aligned} A_r''(t) - \frac{3t}{r^2}A_r'(t) + \frac{3}{r^2}A_r(t) + G\frac{t^2-r^2}{r^3}W_r(t) \\ (\frac{1}{t}W_r'(t) - \frac{3}{r^2}W_r(t)) &= 0 \end{aligned} \tag{27}$$

We proceed by 1) finding initial conditions and numerical solutions to Eq.(26) for W(t) for a series of r-values, 2) fitting a function to these values, and 3) finding the function

$$H(t) = \frac{t^2-r^2}{r^3}W(t)(\frac{1}{t}W'(t) - \frac{3}{r^2}W(t)), \tag{28}$$

which is used in Eq.(27) to obtain an approximate solution for A(t) and thereby A(r,t), using Eq.(25).

In Figure 1 $W_r(t)$ is given for various values of r, and the time for the creation of the bubble wall is clearly seen. In Figure 2 similar results are shown for $A_r(t)$. Note that near the bubble wall A^{em} becomes infinite and has an instanton-like behavior

$$A(r,t)|_{r \simeq r_{wall}} \simeq \frac{A_W}{((r^2-t^2)+\zeta^2)^2} \,. \tag{29}$$

Away from the surface A(t) becomes smaller than this instanton-like solution.

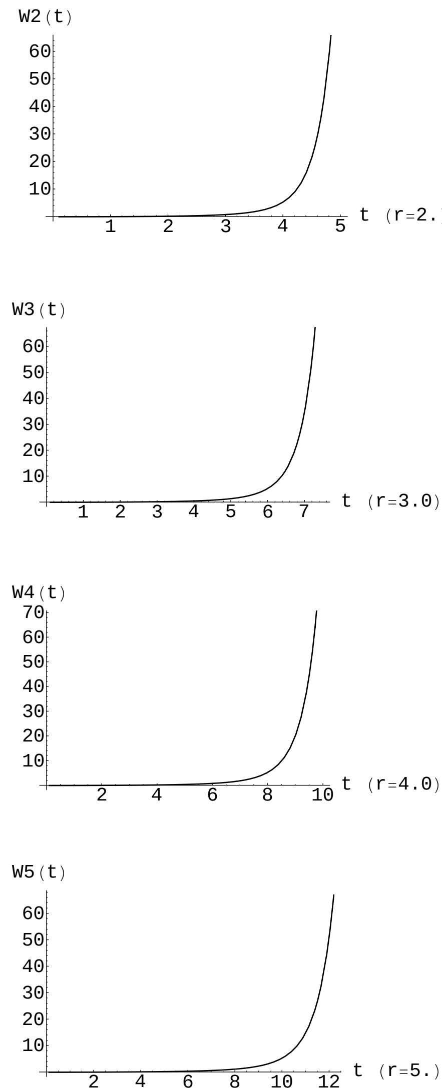

FIG. 1: The function $W_r(t)$ for various values of r

From Figure 2 one observes that the time at which one reaches the radius of the wall bubble is given approximately by

$$t \simeq 2r \,, \tag{30}$$

from which we obtain the nucleation velocity of the bubble wall.

$$v^{wall} \simeq \frac{c}{2} \,. \tag{31}$$

This is an important result in our work on magnetic field generation and evolution, as well as our recent

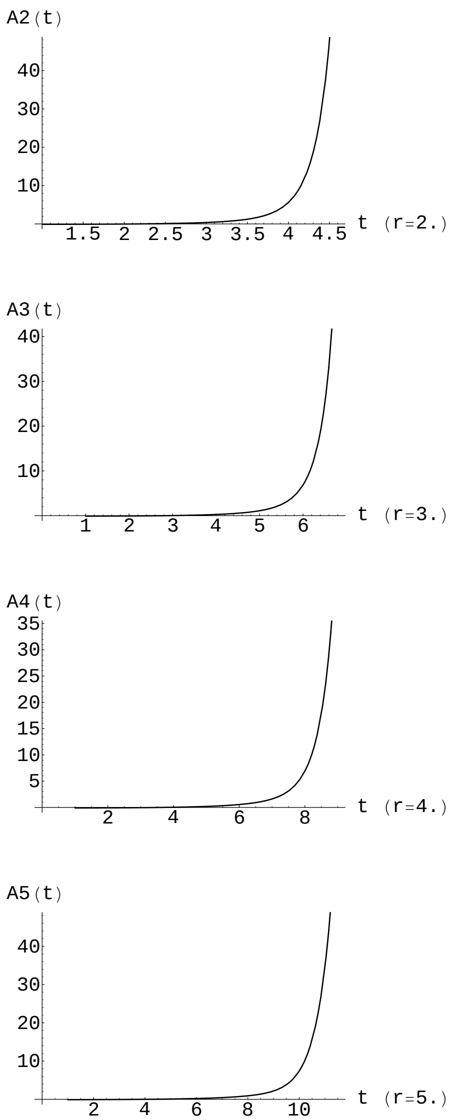

FIG. 2: The function $A_r(t)$ for various values of r

study of gravity wave production by magnetic fields created in the EWPT[24]

In our present work $A^{em}_\nu \sim x_\nu A(r,t)$, so electric fields but no magnetic fields are created. This work can provide the initial conditions for EWPT collisions in which magnetic fields are created.

IV. ELECTROMAGNETIC FIELD CREATION DURING NUCLEATION WITH I-SPIN ANSATZ AND FERMIONS

Let us now return to Eq(14), the e.o.m. for the em field with the electron current as well as the charged gauge fields

$$\partial^2 A^{em}_\nu - \partial_\mu \partial_\nu A^{em}_\mu - \frac{gg'}{\sqrt{g^2+g'^2}}\epsilon^{3jk}\mathcal{W}^{jk}_\nu$$
$$-J^e_\nu \quad = 0 \; ,$$

with the electron current at finite temperature[25]

$$\begin{aligned} J^{lep}_\nu &= G n_e \bar{u}_e \gamma_\nu u_e \equiv j \bar{u}_e \gamma_\nu u_e \\ n_e &= \frac{3}{4\pi^2}\zeta(3) T^3 \; , \end{aligned} \qquad (32)$$

with $\zeta(3) \simeq 1.2$, G=$gg'/\sqrt{g^2+g'^2}$=0.303, and u_e a Dirac spinor.

From Eq(14) we obtain the e.o.m. for $A_r(t)$ (see previous section)

$$A'' - \frac{3t}{r^2}A' + \frac{3}{r^2}A + GH - 3r^2 j \;=\; 0 \; , \qquad (33)$$

with $j = 0.028 M^3_{Higgs}$. We have taken $T = M_{Higgs}$ at the time of the EWPT.

The solutions for $A_r(t)$ for fixed r are shown in Figs. 3 and 4. As one can see, the electron current plays an important role in electromagnetic field production during nucleation of EWPT bubbles. Note that the current of the charged W is approximately -$jW_r(t)$. The solutions for $jW_r(t)$ for fixed r, in comparison to the electron current, are shown in Fig 5. The function $jWr(t)$ should be compared to the electron current = -0.084r^2. Although the electron current is much smaller than the current from the W^+, W^- fields near the bubble surface, the electron current is larger than the W^+, W^- currents in the interior of the bubble..

Since the electron current is larger than the charged gauge currents inside the bubble, as shown in Fig. 5, we conclude that the leptonic currents must be included for the correct derivation of the magnetic fields during EWPT bubble collisions.

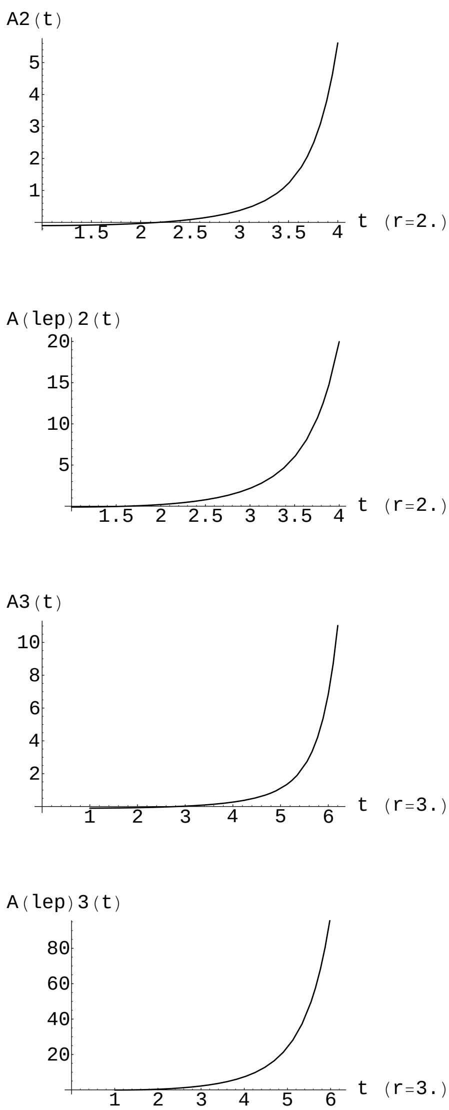

FIG. 3: A(t) without and A(lep)(t) with lepton currents for r=2.0 and 3.0

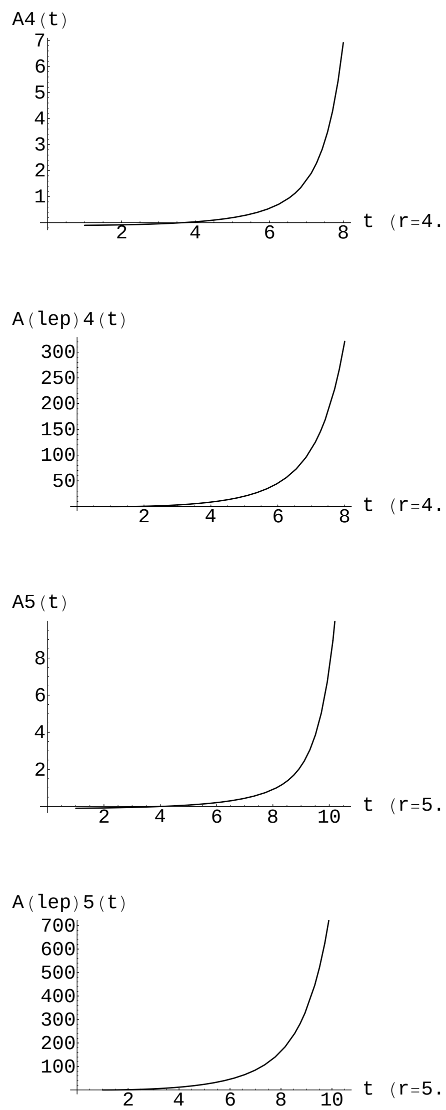

FIG. 4: A(t) without and A(lep)(t) with lepton currents for r=4.0 and 5.0

V. CONCLUSIONS

We have formulated the coupled equations of motion for the electroweak MSSM with a Lagrangian that adds the right-handed Stop field terms to the Standard Model.

In this model a first order EWPT can occur with satisfactory baryogenesis. By using an I-spin ansatz for spherically symmetric bubble nucleation we were able to derive a Maxwell-like equation of motion for the electromagnetic field with the current given by the electrically charged gauge fields, $W^{\pm}$, as well as the charged fermion currents. Moreover, by treating the W_t and W_j components separately we were able to decouple the equations for the $W^{\pm}$ fields from the other gauge fields, and also the Higgs and Stop fields, and obtain the current for the electromagnetic field.

In the present paper we derived solutions for the electromagnetic field caused by EWPT bubble nu-

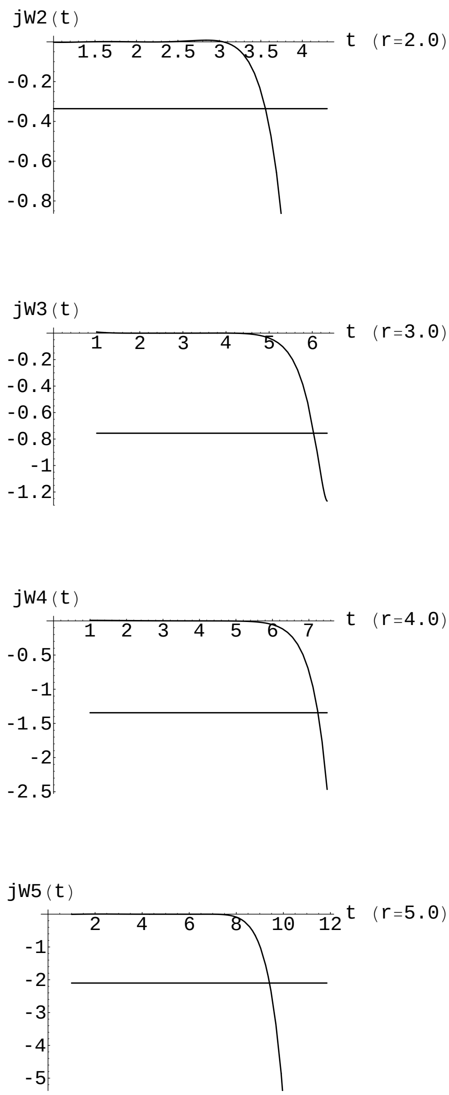

FIG. 5: jWr=W current for r=2.0, 3.0, 4.0, and 5.0 The horizontal lines are the electron current = -0.084 r^2

cleation, using a Coulomb gauge condition to obtain ordinary differential equations, from which we found instanton-like solutions for the electromagnetic field in the region of the bubble wall. We calculated the electromagnetic field produced in the EWPT bubble nucleation with electron currents as well as the $W^{\pm}$ fields, and found that although the electron current did not affect the em field near the bubble wall, they play an important role within the bubble, which could be quite important for bubble collisions.

Although our present work is a very limited physical problem, it explores new physics which can arise from nucleation before collisions starting from a MSSM electroweak Lagrangian with leptons. This will be investigated in a continuation of our current research[21, 22] on magnetic field creation in EWPT bubble collisions. In the future we shall investigate whether the resulting magnetic fields could serve as seeds for gallactic and extra-gallactic large-scale magnetic fields.

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